
Microeconomics I

Class 5 — Indirect utility, wealth and price effects

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Plan for today

Warm-up; rice in Hunan

Indirect utility

Roy's identity

Break

Wealth effects: normal and inferior goods

Price effects: Giffen goods, substitutes, complements

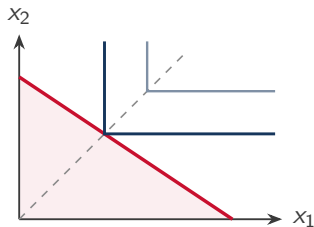
In pairs: from v to demand; summary

Practicalities

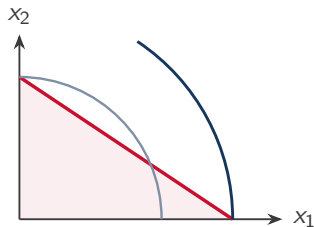
- Problem set 1: due at the start of class tomorrow, Wed 14 Oct, on paper, one copy per group
- Office hours: Wednesdays 14:00–15:00, Room 445
up to 20 minutes per student; those who book by e-mail have priority
- Slides and reading: pkocourek.com/teaching/micro-1 · pkocourek@ub.edu

Warm-up: where does each consumer buy?

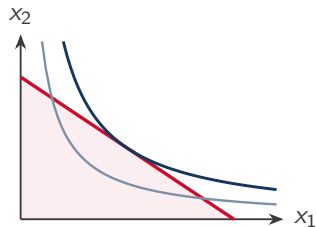
$p = (2, 3)$, $w = 12$ in every panel



L-shaped, kinks on $x_2 = x_1$



bowed out



$x_1 x_2$ or $\ln x_1 + \ln x_2$

Rice gets cheaper. Do poor households eat more rice?

Hunan, China, 2006: 650 poor urban households

rice: 64 % of their calories

at random, vouchers: up to a quarter
off the price of rice, for five months

More rice, the same, or less?

A field experiment; what the households did, in class.

Revealed in class

Before class: decide which answer you expect, and why.

Indirect utility

MWG 3.D, 3.G

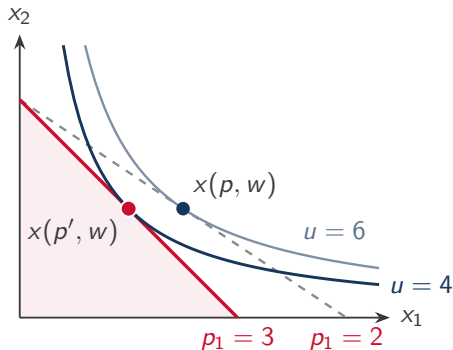
Indirect utility: the level of the best curve the budget reaches

$$v(p, w) = u(x^*) \text{ for any } x^* \in x(p, w)$$

the value of the UMP: $\max\{u(x) : x \in \mathcal{B}_{p,w}\}$

$$u = x_1 x_2: \quad v(p, w) = \frac{w}{2p_1} \cdot \frac{w}{2p_2} = \frac{w^2}{4p_1 p_2}$$

at $p = (2, 3)$: $w^2/24$ (class 4, slide 26)



$p_1: 2 \rightarrow 3$: v falls from 6 to 4.

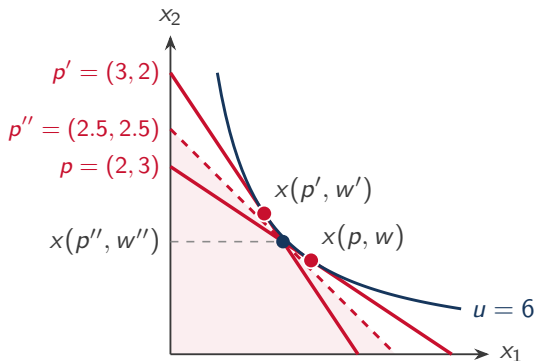
MWG 3.D, p. 56.

$v(p, w)$ is homogeneous, rises with w and never rises with prices

property	why	for $v = \frac{w^2}{4p_1p_2}$
(i) homogeneous of degree 0	the same budget set	$\frac{(\alpha w)^2}{4 \alpha p_1 \alpha p_2} = v$
(ii) strictly increasing in w	LNS (class 2)	$\frac{\partial v}{\partial w} = \frac{w}{2p_1p_2} > 0$
nonincreasing in each p_ℓ	the budget set shrinks	$\frac{\partial v}{\partial p_1} = -\frac{w^2}{4p_1^2p_2} < 0$
(iii) quasiconvex in (p, w)	next slide	
(iv) continuous		

MWG Proposition 3.D.3, p. 56: u continuous, $\succsim$ locally non-satiated, $X = \mathbb{R}_+^2$.

v is quasiconvex: the average of two budgets never beats both



$\{(p, w) : v(p, w) \leq \bar{v}\}$ is convex
for every $\bar{v}$

$p = (2, 3)$ or $p' = (3, 2)$, $w = 12$:
 $v = 6$ both

their average $(2.5, 2.5)$, $w = 12$:
 $v = 5.76$

What the average budget affords, one of the two already affords: $v \leq 6$.

MWG Proposition 3.D.3(iii), p. 56; Figure 3.D.5, p. 57, in Wednesday's reading; $u = x_1 x_2$.

In pairs: could this be an indirect utility function?

Test (i) and (ii) of Prop. 3.D.3 (slide 9). Four minutes.

$v(p, w) =$	homogeneous of degree 0	$\uparrow$ in w , $\downarrow$ in p
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$$\frac{w}{p_1 + p_2}$$

$$\frac{w^2}{p_1 p_2}$$

$$\frac{p_1 + p_2}{w}$$

$$\ln w - \ln p_1 - \ln p_2$$

Answers revealed in class. Try it in pairs before class.

Change the utility function: v and λ change, demand does not

u	$v(p, w)$	$\lambda = \partial v / \partial w$ at $p = (2, 3), w = 12$
$x_1 x_2$	$\frac{w^2}{4p_1 p_2}$	$\frac{w}{2p_1 p_2} = 1$
$\frac{1}{2} \ln x_1 + \frac{1}{2} \ln x_2$	$\ln w - \frac{1}{2} \ln p_1 - \frac{1}{2} \ln p_2 - \ln 2$	$\frac{1}{w} = \frac{1}{12}$

Same $\succsim$, same $x(p, w) = (3, 2)$; v becomes $f(v)$, and λ with it.

MWG p. 57, in Wednesday's reading ($\tilde{u} = f(u)$ gives $\tilde{v} = f(v)$; Example 3.D.2 with $\alpha = \frac{1}{2}$);
 $\lambda = \partial v / \partial w$: pp. 54–55, class 4, slide 26.

Roy's identity: demand from two slopes of v

$$x_\ell(p, w) = - \frac{\partial v(p, w) / \partial p_\ell}{\partial v(p, w) / \partial w}$$

u continuous; $\succsim$ locally non-satiated and strictly convex; v differentiable at $(p, w) \gg 0$

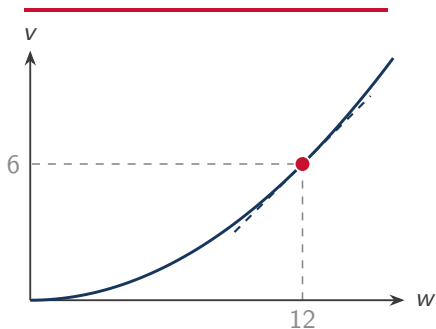
$$v = \frac{w^2}{4p_1p_2}: \quad - \frac{-w^2/(4p_1^2p_2)}{w/(2p_1p_2)} = \frac{w}{2p_1}$$

Cobb–Douglas, class 4

Replace u by $f(u)$: both slopes gain the factor f' , and the ratio does not change.

MWG Proposition 3.G.4 (Roy's identity), p. 73.

A dearer good costs utility in proportion to how much of it she buys

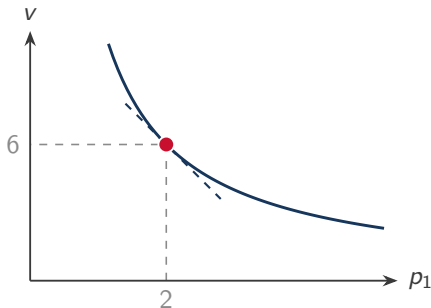


$p = (2, 3)$: slope $\lambda = 1$

$$\frac{\partial v}{\partial w} = \lambda, \quad \frac{\partial v}{\partial p_\ell} = -\lambda x_\ell(p, w)$$

Here $x_1 = -(-3)/1 = 3$: the bundle $(3, 2)$ again.

MWG p. 74, Proofs 2 and 3 (the envelope theorem); $u = x_1 x_2$, $p = (2, 3)$, $w = 12$.



$w = 12$, $p_2 = 3$: slope $-\lambda x_1 = -3$

(optional) A bliss point: does more wealth always raise v ?

$$u(x) = -(x_1 - 2)^2 - (x_2 - 2)^2 \text{ on } \mathbb{R}_+^2, \quad p = (2, 3).$$

(1) $v(p, w)$ for $w \geq 10$.

(2) Which part of Prop. 3.D.3 fails?

Why?

(3) $\lambda = \partial v / \partial w$ at $w = 12$?

At home if not done in class; the answers are in the slides released after class.

MWG Proposition 3.D.3, p. 56 (slide 9).

Break

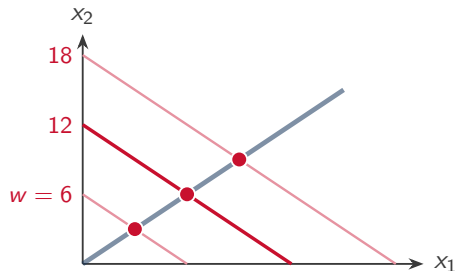
Ten minutes

Wealth and price effects

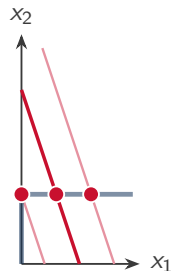
MWG 2.E

More wealth: Cobb–Douglas scales up; quasilinear: all to good 1

Engel function: $w \mapsto x(p, w)$, p fixed; wealth effect $\partial x_\ell / \partial w$



$$u = x_1 x_2, \quad p = (2, 3)$$
$$x = (w/4, w/6): \text{ a ray}$$

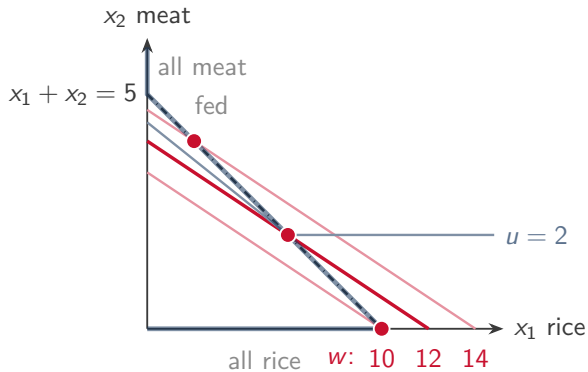


$$u = x_1 + \ln(1 + x_2), \quad p = (3, 1)$$
$$x_2 = 2 \text{ once } w \geq 2$$

Class 4's promise: past $w = 2$, every extra euro goes to good 1.

MWG 2.E, pp. 24–25, Figure 2.E.1 (the wealth expansion path).

Rice can be inferior: past subsistence, more wealth buys less of it



$$p = (2, 3)$$

she needs 5 units of food:

$$x_1 + x_2 \geq 5$$

once fed, only meat counts

normal at (p, w) : $\partial x_\ell / \partial w \geq 0$;

inferior: < 0

Rice is normal up to $w = 10$, inferior from 10 to 15.

$u = \min\{x_2, x_2 - 4(5 - x_1 - x_2)\}$: meat, minus 4 per unit of missing food. MWG 2.E, p. 25; own example.

The budget ties the effects together: not every good can be inferior

$$\sum_k p_k \frac{\partial x_\ell}{\partial p_k} + w \frac{\partial x_\ell}{\partial w} = 0$$

homogeneity: Prop. 2.E.1

$$\sum_\ell p_\ell \frac{\partial x_\ell}{\partial w} = 1$$

Walras in w : Engel aggregation, Prop. 2.E.3

$$\sum_\ell p_\ell \frac{\partial x_\ell}{\partial p_k} + x_k = 0$$

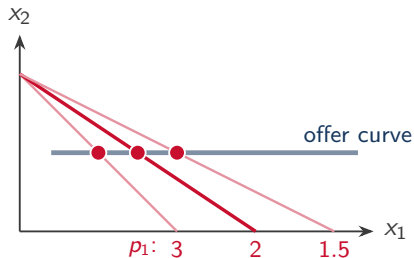
Walras in p_k : Cournot aggregation, Prop. 2.E.2

Slide 19 at $w = 12$: $2 \cdot (-1) + 3 \cdot (+1) = 1$. An extra euro is spent somewhere.

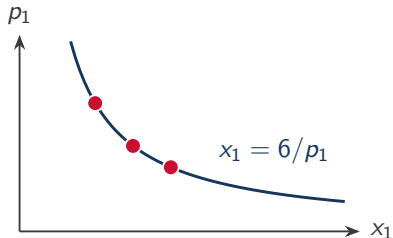
MWG 2.E, pp. 27–28.

Cheaper good 1: Cobb–Douglas buys more of it, as much of good 2

usual: $\partial x_\ell / \partial p_\ell \leq 0$; Giffen at (p, w) : $\partial x_\ell / \partial p_\ell > 0$



goods: the offer curve



prices: the demand curve

$x_1 = 6/p_1$ rises as p_1 falls; x_2 stays at 2: the offer curve is flat.

MWG 2.E, pp. 25–26, Figures 2.E.2–2.E.3; $u = x_1 x_2$, $w = 12$, $p_2 = 3$.

Revealed in class

Before class: on slide 19's picture, let rice cost 1.5 instead of 2
and find her new best bundle.

Dearer good 2: x_1 falls, stays or rises, depending on σ

gross substitutes: $\partial x_1 / \partial p_2 > 0$; gross complements: < 0

CES: $u = (\frac{1}{2}x_1^\rho + \frac{1}{2}x_2^\rho)^{1/\rho}$, $\sigma = 1/(1 - \rho)$; $\sigma = 2$: same $\succsim$ as $\sqrt{x_1} + \sqrt{x_2}$

σ	u	$x_1(p, w)$	$p_2: 3 \rightarrow 4$ $p_1 = 2, w = 12$
0	$\min\{x_1, x_2\}$	$\frac{w}{p_1 + p_2}$	$2.4 \rightarrow 2$: complements
1	$x_1 x_2$	$\frac{w}{2p_1}$	$3 \rightarrow 3$: neither
2	$\sqrt{x_1} + \sqrt{x_2}$	$\frac{w p_2}{p_1(p_1 + p_2)}$	$3.6 \rightarrow 4$: substitutes

CES: complements for $\sigma < 1$, neither at $\sigma = 1$, substitutes for $\sigma > 1$.

MWG 3.G, p.70 (“gross” = Walrasian demand; the Hicksian version is Wednesday). CES: class 3, slide 26.

In pairs: Roy's identity, then the three effects

Find the demand by Roy's identity. Wealth effect? Own price: usual or Giffen?

The other price rises: substitutes or complements? Seven minutes.

$v(p, w)$	demand	wealth	own price	other price $\uparrow$
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$$\frac{w^3}{p_1 p_2^2}, \text{ find } x_1$$

$$\frac{w + p_2}{p_1} - 1 + \ln \frac{p_1}{p_2},$$

find x_2 class 4's $x_1 + \ln(1 + x_2)$, $p_1 > p_2$

$$\frac{w - 5p_1}{p_2 - p_1}, \text{ find } x_1$$

slide 22's consumer, near $p = (2, 3)$, $w = 12$

Answers revealed in class. Try it in pairs before class.

(optional) Good 1 is Giffen: what happens to x_2 when p_1 rises?

A consumer with two goods; Walras' law holds; $\partial x_1 / \partial p_1 > 0$.

(1) Show that $\partial x_2 / \partial p_1 < 0$.

Hint: slide 20, line 3, with $k = 1$.

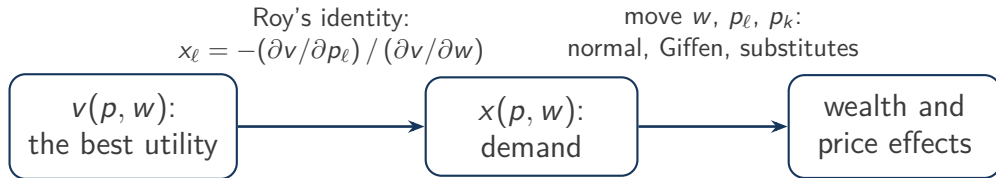
(2) Explain (1) with her spending.

(3) Can $\partial x_1 / \partial p_2 > 0$ hold too?

At home if not done in class; the answers are in the slides released after class.

MWG 2.E, pp. 27–28 (Proposition 2.E.2, Cournot aggregation).

Today in one line



Wednesday: expenditure minimisation — Hicksian demand, Shephard's lemma, duality.

Problem set 1: due at the start of class tomorrow, on paper, one copy per group.

Problem set 2 is handed out tomorrow; due in class Wednesday 21 October.

Read MWG 3.E–3.G (or Muñoz-Garcia ch. 2) for Wednesday 14 October.